

Math 10

Lesson 3–5 Graphs of Relations and Functions

I. Lesson Objectives:

- 1) Deepening our understanding of the meaning of domain and range.
- 2) Learning about the vertical line test for a function.
- 3) Understanding the difference between discrete and continuous data sets.
- 4) Expressing domain and range in a variety of ways.

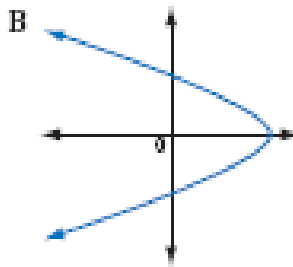
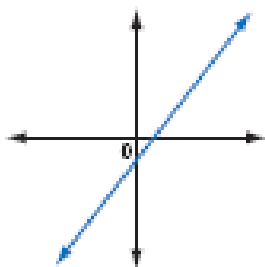
II. Relation and function revisited

Recall that a **function** is a special type of relation where for any value of x there is only one value of y . In other words, for every value in the domain there is a unique value in the range. Further, while all functions are relations, not all relations are functions.

Example 1 Determining whether a relation is a function

For each pair of relations, decide which relation is a function and which is not a function. Explain your choice.

a) A



b) C $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

D $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$

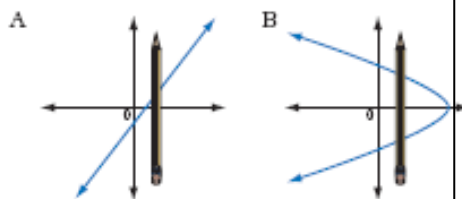
Solution

For each pair, check to see if the relation has a domain value associated with more than one range value.

- a) When a relation is given as a graph, look for any points on the graph that line up vertically. If two or more points line up vertically, the relation is not a function.

One method to check this is called the **vertical line test**. If any vertical line intersects the graph at more than one point, the relation is not a function. In the

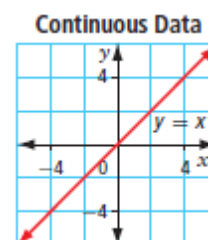
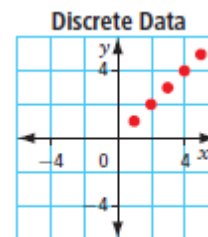
example to the right, relation A is a function, because it passes the vertical line test. Each domain value is used only once. Relation B does not pass the vertical line test and is not a function.



- b) In relation C, each domain value is used only once, so it is a function. In relation D, the domain value 1 is used repeatedly. Therefore, relation D is not a function.

III. Discrete and Continuous Data

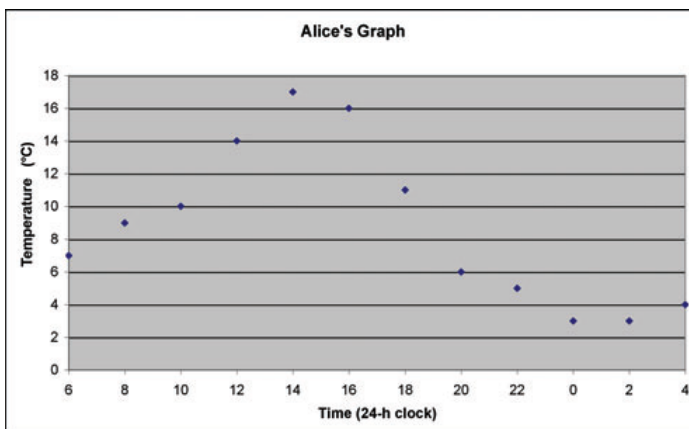
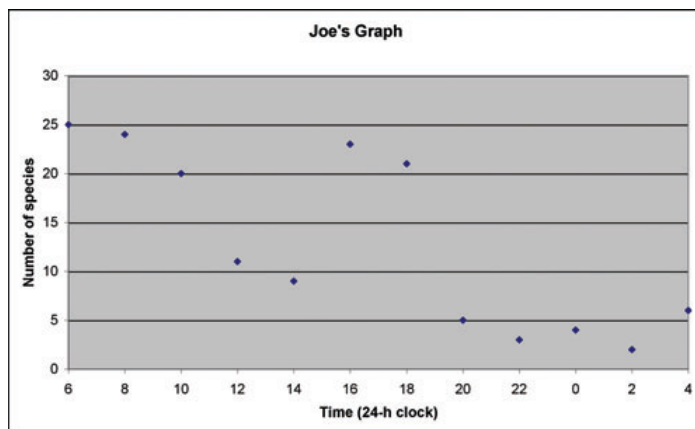
Discrete data are when data values on a graph are not connected. A graph of discrete data only shows points because the values in between have no meaning. For **continuous** or **non-discrete** data the values on a graph are connected. A graph of continuous data is a solid line or curve. For example, say a relation is defined by the set of ordered pairs $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$. These are **discrete** data and the graph has five unconnected points.



However, for the relation defined by the equation $y = x$, there are an infinite number of possible ordered pairs – i.e. for any x -value there is a corresponding y -value. The points $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$, and $(5, 5)$ satisfy this relation and so do many other points such as $(3/2, 3/2)$, $(-3.6, -3.6)$ and $(\sqrt{2}, \sqrt{2})$. On a graph you show an infinite set with an unbroken, or **continuous**, line or curve.

Question 1

In an environmental study in Northern Alberta, Joe collected data on the number of different species of birds he heard or saw in a one hour period every two hours for 24 h. Alice collected data on the temperature in the area at the end of each two hour period. They plotted their data:



a) Does each graph represent a relation? A function? How can you tell?

b) Which of these graphs should have the data points connected? Explain.

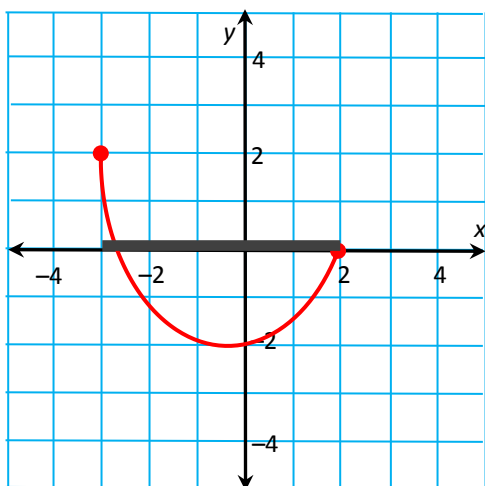
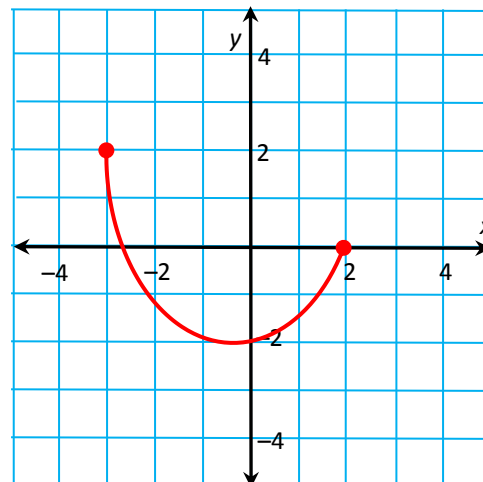
IV. Determining the domain and range of a function / relation

Recall from Lesson 3-2 the concept of domain and range.

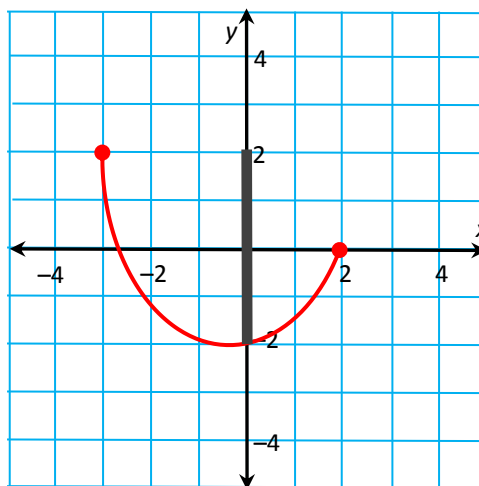
- The **domain** of a relation is the set of all possible values for the *independent* variable. In other words, the domain is the *x*-values.
- The **range** of a relation is the set of all possible values for the *dependent* variable. In other words, the range is the *y*-values.

In a set of ordered pairs, values for the domain are the first element of each pair. Values for the range are the second element. On a graph, values of the domain are plotted along the horizontal axis and values of the range are plotted along the vertical axis.

To determine the domain and range of a relation we have to be able to correctly interpret the graph. Consider the graph to the right. The dot at the end of the graph indicates that the graph stops at that point. A solid dot $\bullet$ at the end of a graph indicates that the end point is included in the set, while an open dot $\circ$, indicates that the point is not included.

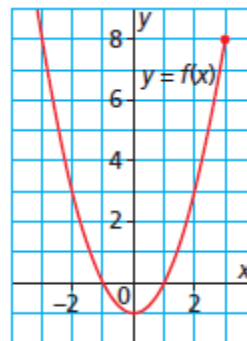


The **domain** is all of the *x*-values that the relation can have. In this case, the first *x*-value is -3 and the final *x*-value is 2 . *x* can be any value between, and including, -3 and 2 . Therefore the domain is $-3 \leq x \leq 2$ that we say as “*x* is greater than or equal to -3 and less than or equal to 2 .”



The **range** is all of the *y*-values that the relation can have. The first *y*-value is -2 and the final *y*-value is 2 . Therefore the range is $-2 \leq y \leq 2$ that we say as “*y* is greater than or equal to -2 and less than or equal to 2 .”

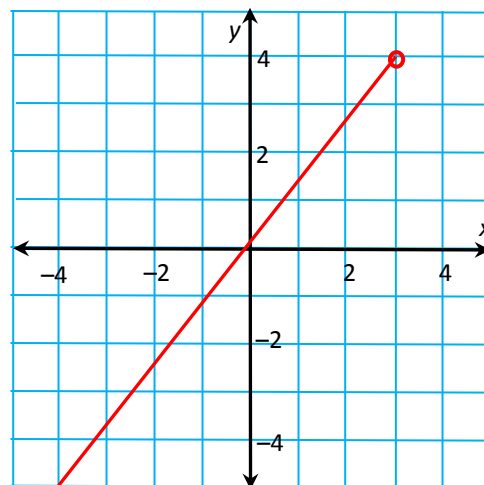
For the function shown to the right the dot at the right end of the graph indicates that the graph stops at that point. There is no dot or circle at the left end of the graph, so the graph continues on to infinity.



The domain is the set of all real numbers that are less than 3, that is $x < 3$.

The range is the set of all real numbers that are greater than -1 , that is $y \geq -1$.

As a final example, for the function shown to the right the circle at the right end of the graph indicates that the graph stops at that point but does not include that point. There is no dot or circle at the left end of the graph, so the graph continues on to infinity.



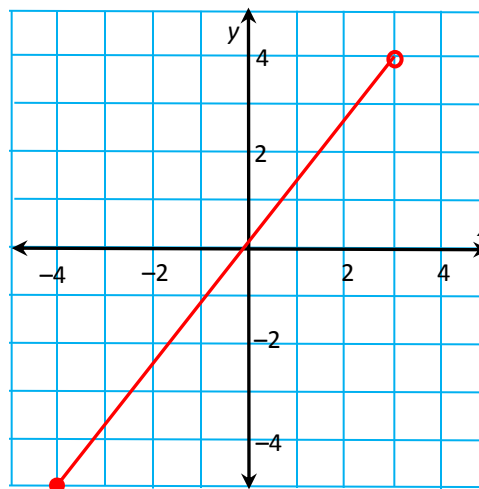
The domain is the set of all real numbers that are less than, but not including, 3, that is $x < 3$.

The range is the set of all real numbers that are less than, but not including, 4, that is $y < 4$.

V. Expressing the domain and range of a function / relation

There are a variety of ways to express the domain and range of a relation. Using the graph as an example:

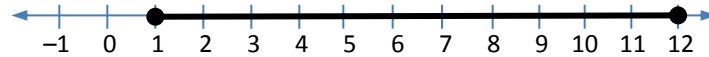
- Using a **word description**:
The domain is the set of all real numbers between -4 and 3 that includes -4 but excludes 3 . The range is the set of all real numbers between -5 and 4 , that includes -5 but excludes 4 .
- Using **set notation**:
Domain $-4 \leq x < 3$
Range $-5 \leq y < 4$



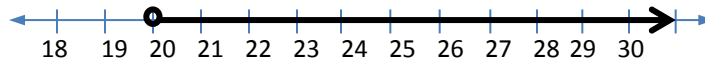
Set notation and word description are the two most common ways to describe domain and range. However, other contexts may require a different description.

- For **discrete data**, a simple list is a useful way to give the domain and range. For example, in the relation $(0, 0), (1, 5), (3, 7), (5, 7)$, the domain is $\{0, 1, 3, 5\}$ and the range is $\{0, 5, 7\}$.

- When describing the numbers on a single line, a **line number** description is useful. For example, this number line represents all numbers between, and including, 1 and 12.



This number line represents all numbers greater than 20.



The open circle at 20 indicates that the value 20 is not included. The arrow pointing to the right indicates that there is no upper limit.

- Interval notation** uses different brackets to indicate the domain/range.
 A square bracket, $]$, is used if the end or beginning number is **included**.
 A soft bracket, $)$, is used if the end or beginning number is **not included**.
 The infinity symbol, ∞ , is used if there is no end point.
 In the line examples above, the domains would be $[1, 12]$ and $(20, \infty)$.

Question 2

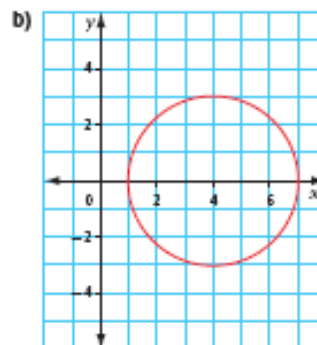
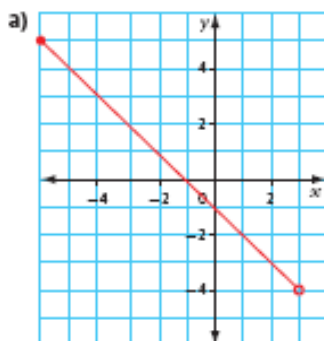
Caitlin is marking time for some music by clapping on the first beat of every bar. The table of values and the set of ordered pairs show the relationship between the total number of beats and her total number of claps. Give the domain and range of the relation using words and a list.

Number of Beats	Number of Claps
1	1
5	2
9	3
13	4
17	5
21	6
25	7
29	8

Question 3

For each graph:

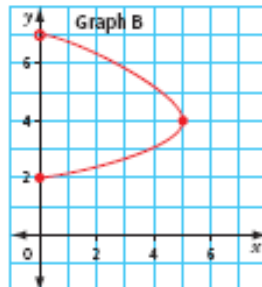
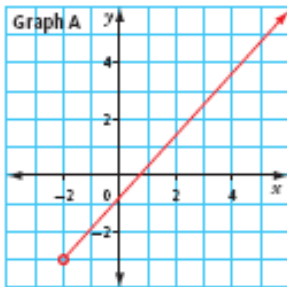
- Does the graph describe a relation?
- Is it a function?
- Give the domain and range using words, interval notation, and set notation.



Question 4

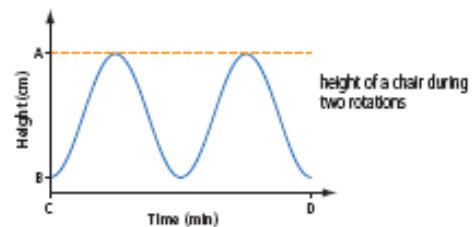
For each graph:

- A) Does the graph describe a relation?
- B) Is it a function?
- C) Give the domain and range using words, interval notation, and set notation.



Question 5

A motorized model Ferris wheel has a radius of 22 cm. The support structure keeps the bottom of the wheel 3 cm above the base. It takes 10 s to complete one revolution. The graph shows the height of one of the chairs during two rotations of the wheel, starting at the lowest point.



- a) What are the values of A, B, C, and D? What do they represent?
- b) What are the domain and range of the graph? Express each in words, as a number line, in interval notation, and in set notation.

Question 6

The data for a relation are recorded in the table of values. Give the domain and range using set notation and lists.

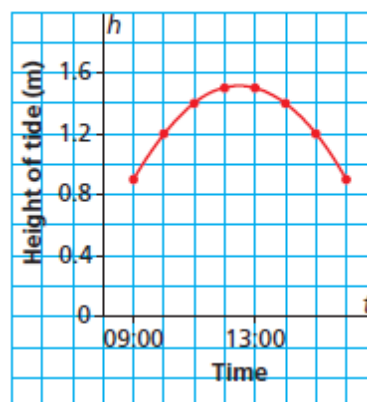
a	b
-3	5
-2	6
-1	7
0	8
1	9
2	10

Question 7

This graph shows the approximate height of the tide, h metres, as a function of time, t , at Port Clements, Haida Gwaii on June 17, 2009.

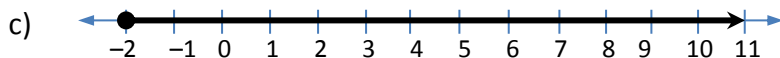
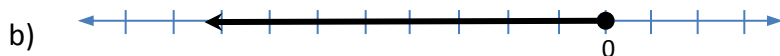
- Identify the dependent variable and the independent variable. Justify your choices.
- Why are the points on the graph connected? Explain.
- Determine the domain and range of the graph.

Height of Tide at Port Clements,
June 17, 2009

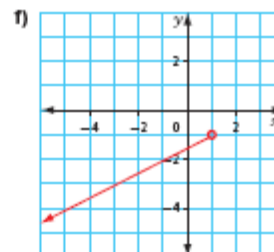
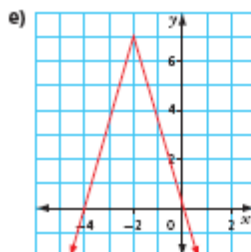
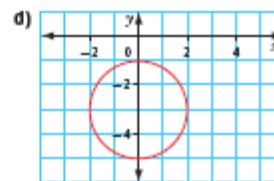
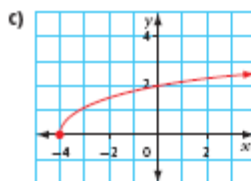
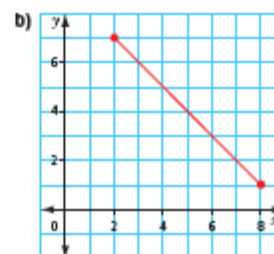
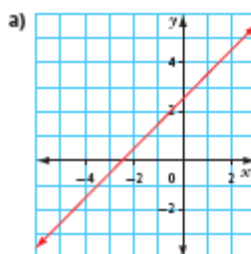


VI. Assignment

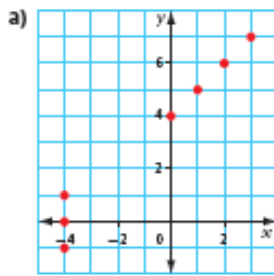
- Describe the set of numbers indicated by each number line. Use words, interval notation, and set notation.



- Give the domain and range of each graph. Use words, a number line, interval notation, and set notation.



3. Write the domain and the range of each relation.



b)

s	v
-4	5
-2	5
0	7
2	7
4	9
6	9

c) (50, 10), (100, 20), (150, 30), (200, 40)

4. A relation is given by the formula $k = 2.8m - 3.5$.

- If the domain of the relation is $[0, 25]$, what is the range?
- Graph the relation on a graphing calculator. Record the window values you chose. Sketch the relation.

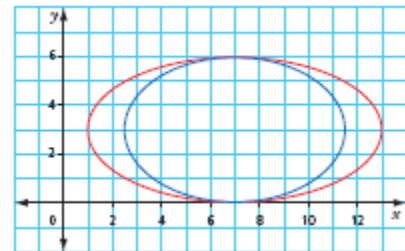
5. The table gives the average annual high temperature for a number of western and northern Canadian cities.

- Give the domain and the range for the relation.
- Graph the relation.

City	Average Annual High Temperature ($^{\circ}\text{C}$)
Winnipeg	8.3
Regina	9.1
Edmonton	8.5
Calgary	10.5
Vancouver	13.7
Victoria	14.1
Whitehorse	4.5
Yellowknife	-0.2

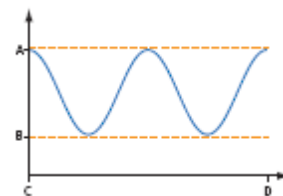
6. A company offers two models of above-ground oval swimming pools. The ovals in the graph are drawn using the dimensions, in metres, of the pools.

- Using interval notation, what are the domain and range of the smaller oval?
- Using set notation, what are the domain and range of the larger oval?
- What are the actual dimensions of each pool?

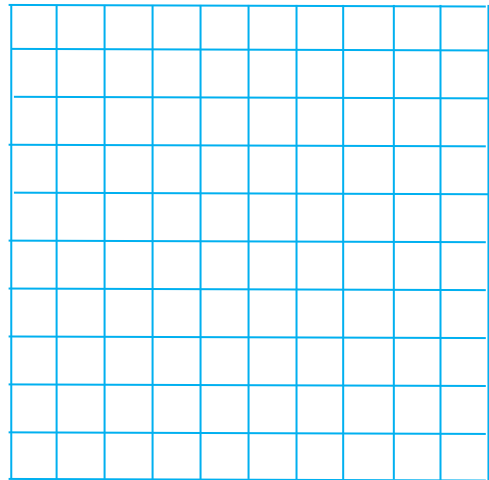


7. The graph shows the changes in tide levels for Bella Coola on the central coast of British Columbia for a 24-h period starting at 12:00 a.m. The greatest water depth, at high tide, is 15.9 ft. The least depth, at low tide, is 4.5 ft.

- Label each axis with appropriate dependent and independent variables.
- What are the values of A, B, C, and D? What do they represent?
- What are the domain and range of the graph? Express these in words, as a number line, in interval notation, and in set notation.

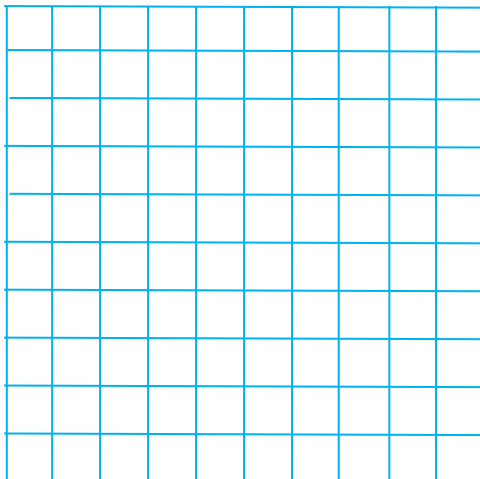


8. A hot-air balloon is flying at an altitude of 1236 m. It begins to descend at a rate of 10 m per minute.
- How long does it take the balloon to reach the ground?
 - Assign variables to represent each quantity in the relation. Identify the independent variable and the dependent variable. Graph the balloon's progress from the moment it begins its descent until it reaches the ground.
 - Does the graph continue in both directions? Explain.
 - What do the domain and range represent in this context?
 - Give the domain and the range using all forms appropriate for this situation.



9. Paulatuuq is north of the Arctic Circle. The table shows the number of hours, h , the sun is above the horizon every 60 days from January 1st, which is day 0.
- Identify the independent variable and the dependent variable. Justify your choices.
 - Graph the data in the table. Did you connect the points? Why or why not?
 - Use the table of values and the graph to explain why this relation is a function.

Day	h
0	0
60	9.7
120	18.5
180	24.0
240	15.9
300	7.4
360	0



10. One litre of latex paint covers approximately 8.5 m^2 and costs \$12.

- Complete this table.
- Graph the area covered as a function of the volume of paint.
- Graph the area covered as a function of the cost.
- Write the domain and range of the functions in parts b and c.

Volume of Paint, p (L)	0	2	4	6	8
Cost, c (\$)	0	24			
Area Covered, A (m^2)	0	17			

